An anti-aliasing POCs interpolation method for regularly undersampled seismic data using curvelet transform

Hua Zhang a,⁎, Hengqi Zhang a, Junhu Zhang a, Yaju Hao a, Benfeng Wang b,⁎

a State Key Laboratory of Nuclear Resources and Environment, East China University of Technology, Nanchang 330013, China
b State Key Laboratory of Marine Geology, School of Ocean and Earth Science, Institute for Advanced Study, Tongji University, Shanghai 200092, China

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A B S T R A C T
Seismic data interpolation are considered the key step in data pre-processing. Most current interpolation methods are just suitable for random undersampled cases. To deal with regular undersampled issue, we propose a novel anti-aliasing Projection Onto Convex Sets (POCS) interpolation method using the curvelet transform. First, we decompose the curvelet transform into two operators: a frequency-wavenumber (f-k) operator and a curvelet tiling operator. These two operators are used to respectively link time-space (t-x) domain to f-k domain, and f-k domain to the curvelet coefficients. In the f-k domain, the two boundaries for dominant dips can be identified by an angular searching within the whole frequency range. Second, we expand the two boundary dips to design a mask function that can eliminate the wraparound aliasing artefacts caused by regular undersampling. Finally, by incorporating the mask function into conventional POCS method, we are able to derive a robust anti-aliasing POCS interpolation method under the curvelet transform. With an exponential threshold model, the satisfactory interpolation result can be obtained by 10–12 iterations. The proposed interpolation method, which has no assumption for linear or quasi-linear events like a Fourier transform-based interpolation method, works for either regularly or randomly undersampled seismic data. Synthetic and real data examples are provided to illustrate the performance of the proposed method.

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1. Introduction
In seismic exploration, the recorded data are often randomly and regularly undersampled due to geographic limitation, logistic condition and other unexpected situations preventing to conduct an ideal data acquisition, leading to severe spatial aliasing (Trad, 2009; Zhao et al., 2013; Gan et al., 2015; Bai et al., 2018; Chen et al., 2019). Also note that even a perfect seismic data acquisition was conducted, its sampling rate could still not be sufficient to perform some tasks of seismic waveform processing (Hennenfent and Herrmann, 2008; Zu et al., 2016; Zhao et al., 2017). In summary, this spatial undersampling adversely affects later processing steps such as multiple suppression, wave equation migration and inversion etc. (Herrmann, 2010; Ely et al., 2015; Kutsche and Verschuur, 2016; Bai et al., 2016; Chen, 2018). To alleviate the adverse impact of undersampling on those subsequent processing tasks, one direct way is to re-sample densely in the field acquisition stage. Obviously, it is not realistic, from the point view of cost, to solve the undersampled problem by re-sampling seismic data. Alternatively, the undersampling aliased issue can be dealt with interpolation method. On the other hand, if we have an efficient interpolation method, during the acquisition stage, we can plan to miss some traces actively to reduce the economic costs according to the field obstacles, then recovering the missing traces using the interpolation method.

At present, different kinds of interpolation methods have been suggested. Many of these methods are categorized as wave-equation methods (Ronan, 1987; Fomel, 2003) and signal processing methods. Signal processing methods, including prediction error filter methods (Spitz, 1991; Naghizadeh and Sacchi, 2007), rank reduction methods (Kreimer et al., 2013; Ma, 2013; Chen et al., 2016a, 2016b; Zhang et al., 2017), Fourier transform methods (Liu and Sacchi, 2004; Jin, 2010), wavelet transform (Liu and Chen, 2019), curvelet transform methods (Hennenfent et al., 2010; Naghizadeh and Sacchi, 2010), dreamlet transform methods (Wang et al., 2015), machine learning methods (Yu et al., 2016; Jia and Ma, 2017; Siahfar et al., 2017; Wang et al., 2019), have been widely used for their simplicity and high efficiency. However, most of above mentioned methods are not suitable for regularly undersampled seismic data or severe missing traces where the spatial aliasing appear. In order to address the issue, some Fourier transform-based anti-aliasing interpolation methods have been proposed. Gulunay (2003) has introduced the f-k equivalent of f-x interpolation methods by creating a mask function from low frequencies to interpolate aliased data. Xu et al. (2005, 2010) developed an antileakage Fourier transform (ALFT) method, but it may fail to work when the
input data has severe aliasing. Zwartjes and Sacchi (2007) have combined Gulunay’s f-k interpolation method with sparse Fourier inversion to interpolate aliased seismic records. Based on these ideas, Naghizadeh and Sacchi (2007) also proposed the multistep auto regressive reconstruction of seismic records. Schonewille et al. (2009) utilized the dip information of the low frequencies in the f-k domain to introduce an antialias ALFT interpolation method. Curry (2010) has introduced an f-k interpolation method, using a Fourier radial adaptive thresholding strategy, in an attempt to utilize the continuity of events along the frequency axis. Naghizadeh (2012) has introduced an F-k domain method that utilizes information from all desired frequencies for interpolation of aliased seismic data. Chiu (2014) have also proposed the multidimensional interpolation method to address the aliasing problem using a model-constrained minimum weighted norm. However, all of these methods discussed above are based upon the Fourier transform, and they need to assume that the seismic data consist of linear or quasi-linear events. Even though they can handle curved events by deploying a spatial windowing strategy with proper overlapped areas in practice, it is always unreachable to get the optimal parameters if data window cannot be applied successfully, in particular when data contain the large gaps due to missing traces.

To overcome this problem, the curvelet transform is employed as an interpolation kernel because it can provide an essentially optimal representation of wave fields by virtue of its anisotropic curvelet shape. Curvelet-based interpolation methods can indeed easily cope with strong variations in dips. It is a clear advantage with respect to interpolation via Fourier bases (Shahidi et al., 2013; Liu et al., 2015). However, in their papers, these authors have also reported that curvelet-based

Fig. 1. Graphical representation of dominant energy and aliasing in the f-k domain.

Fig. 2. Synthetic seismic data and its f-k spectrum. (a) Synthetic seismic data. (b) 50% regularly missing data. (c-d) The f-k spectra of (a-b), respectively.
Fig. 3. Interpolated result of synthetic seismic data. (a) Interpolated data using the conventional POCS method. (b) Interpolated data using the proposed method. (c-d) The f-k spectra of (a-b), respectively. (e) and (f) are difference plots at the same scale as (a) and (b).
recovery performs less well in the presence of strong coherent aliasing caused by regular undersampling. In order to mitigate this problem, Naghizadeh and Sacchi (2010) proposed the beyond aliasing hierarchical scale curvelet interpolation by creating a mask function. In the curvelet domain, the mask function in the aliasing-free scales is first established using a thresholding strategy, then it is up-scaled to estimate the mask function in the aliased scales. Even though this method can handle coherent aliasing caused by regular undersampling. It is impractical to get the optimal parameters due to the fact that its implementation is too complicated and also costs significant amount of computation time. Thus, it is necessary to study more robust anti-aliasing interpolation algorithms.

POCS algorithm is widely used because of its simplicity and robustness. Abma and Kabir (2006) firstly applied the POCS algorithm to irregular seismic data interpolation, then a number of POCS-based interpolation methods have been developed (Yang et al., 2012; Zhang et al., 2015). However, all currently used POCS methods lack an anti-aliasing mechanism, and therefore fail to recover the regularly missing traces because regularly missing data produce high-amplitude aliasing artefacts in the f-k domain. In this paper, by carefully understanding well-established beyond-aliasing interpolation method (Naghizadeh, 2012) in conjunction with the curvelet transform, we propose an anti-aliasing POCS (AAPOCS) method to interpolate regularly undersampled seismic data, but it can be also applicable to process the randomly undersampled seismic data. The paper is organized as follows. First, by reviewing the properties of the curvelet transform, we decompose the curvelet transform into the f-k operator and the curvelet tiling operator. Second, we introduce the theory of the anti-aliasing POCS method using the curvelet transform. Next, we compare the interpolation performance of the proposed method with conventional POCS method using synthetic and real example, followed by discussions and conclusions.

2. Theory

2.1. Curvelet transform

The curvelet transform is a multiscale, multidirectional, and localized transform. Candès et al. (2006) proposed the fast discrete curvelet transform (FDCT) via wrapping specially selected Fourier samples (FDCT via wrapping). The main steps of FDCT via wrapping are as follows: (1) applying the forward 2D FFT (f-k) to obtain the Fourier coefficients; (2) forming angular wedges and wrapping each wedge around the origin in the Fourier domain; and (3) applying the inverse f-k to each wedge to get the curvelet coefficients. The forward FDCT \( C \) can be defined by

\[
C \overset{def}{=} QF
\]  

(1)

Fig. 4. Recovered SNRs with iterations for synthetic example.

\[
\text{Fig. 5. The distribution of energy for the dip range } (-5 \leq p \leq 5), \text{ computed from the f-k domain of Fig. 2d.}
\]

\[
\text{Fig. 6. The f-k domain mask function obtained from two expanded boundary peaks in Fig. 5.}
\]

Fig. 5. The distribution of energy for the dip range \((-5 \leq p \leq 5)\), computed from the f-k domain of Fig. 2d.

where \( F \) is the forward f-k operator constructed in step 1. \( Q \) is the curvelet tiling operator that links the f-k domain to the curvelet coefficients and combines steps 2 and 3.

2.2. Identifying boundary dips in the f-k domain

Suppose \( d(t,x) \) is the data in the t-x domain and \( D(\omega,k) \) is its f-k spectra of data. The first step of the proposed method is an angular search over a range of dips in the f-k domain to identify the boundary of dominant dips. The origin of the angular rays is located on the origin of the f-k domain \((\omega_0,k_0) = (0,0)\). Due to the symmetrical property of the frequency axis in the f-k domain, we only use the positive frequencies to explain the methodology. Also, for theoretical simplicity, we use the concept of normalized frequencies and wavenumbers. This leads to the ranges \( 0 < f < 0.5 \) for normalized frequencies and \(-0.5 < k < 0.5 \) for normalized wavenumbers. A map of dominant dips is produced by summation along the angular rays:

\[
E(p) = \sum_{n=1}^{N} D(\omega_n, k = p\omega_n - \lfloor p\omega_n + 0.5 \rfloor)
\]

(2)

where \( p \) is the slope of the summation path in the f-k domain. The parameter \( n \) represents the index of normalized frequency and can include any interval of frequencies. The operator \( \lfloor \cdot \rfloor \) denotes the nearest smaller integer value. Notice that eq. (2) also allows the raypath to wrap around the frequency axis to account for aliased data in the f-k domain. These aliased dips are defined by \( p > 1 \) and \( p < -1 \).

The peak values in the function \( E(p) \) are indicators of the dips or slopes with dominant energy. Several peak values can be identified in \( E(p) \). However, we are concerned about two boundary peak values on
the left and right side of function $E(p)$, and we think the values between two peak values are dominant energy of original data. Let us assume that we have been able to identify two boundary slopes with the values $p_{L_1}$ and $p_{L_2}$, and subscript $L_1$ and $L_2$ is the index of slope $p$. In order to preserve small spectral values around the two boundary dips, we need to appropriately expand the two boundary dips to some extent. Let us assume we have two expanded boundary slopes with the values $p_{L_1-L_a}$ and $p_{L_2+L_b}$. $L_a$ and $L_b$ are the expanded length for the left boundary and right boundary slopes, respectively.

2.3. Building a mask function

After identifying the two expanded boundary dips of function $E(p)$, we deploy straight lines along the correspondent angles in the f-k domain, and get the dominant energy area between two expanded boundary dips, as shown in Fig. 1. The goal here is to transfer Fig. 1 into a 2D mask function $H$ with the size of the original data in the f-k domain. 2D mask function $H$ has values of one along the desired angles and zero elsewhere in the f-k domain. In other words, in aliasing area, values are zero. In dominant energy area, the expression is given by

$$H = \begin{cases} 1 & D(\omega_n : \omega_n, k = p_j\omega_n - p_j\omega_n + 0.5) > \tau \quad n = 1, 2, ..., N \\ 0 & D(\omega_n : \omega_n, k = p_j\omega_n - p_j\omega_n + 0.5) < \tau \quad j = L_1 - L_a, L_2 + L_b \end{cases}$$

(3)

Where the threshold $\tau$ is user-specified. We can set $\tau$ to be the value that can preserve our desired coefficients in dominant energy area. The designed mask function can serve to eliminate the unwanted artefacts and preserve the original signal. The dip-search strategy explained here can be easily incorporated into POCS to create the AAPOCS interpolation method.

2.4. POCS method

Abma and Kabir (2006) introduced the POCS method for seismic data interpolation in f-k domain, and the iteration process can be

![Regularly undersampling data and the f-k spectra. (a) Data decimated by a factor of 4. (b) Data decimated by a factor of 6. (c-d) The f-k spectra of (a and b), respectively.](image)
implemented in the curvelet domain to improve the interpolation effect. Its main idea is that the seismic data are transferred to the curvelet domain by the curvelet transform in each iteration and then a threshold is applied to the transformed data, leaving only the highest amplitudes. The inverse curvelet transform is applied to data with events above the threshold amplitude level in the curvelet domain and finally the original traces that do not need to be interpolated are reinserted into the inverted transformed data. Conventional POCs interpolation method in curvelet domain (Zhang et al., 2015) is given by

\[
d^i(t, x) = y^{obs}(t, x) + [I - S(t, x)]C^iTQFQ^+^{-1}(t, x), \quad i = 1, 2, ..., M
\]  

(4)

where \(d^i(t, x)\) represents the 2D interpolated data at the \(i\)-th iteration, \(y^{obs}\) is the original seismic data which satisfies the condition \(y^{obs} = d^0(t, x)\), \(C^i\) represents the inverse 2D curvelet transform. The operator \(S(t, x)\) is the sampling operator where \(S(t, x) = 1\) for observed traces and \(S(t, x) = 0\) for unrecorded traces. We also define the curvelet coefficients \(c^i\) as \(c^i = C^iTQFQ^+^{-1}(t, x)\) at the \(i\)-th iteration. The iteration threshold operator \(T^i\) is given by

\[
T^i = \begin{cases} 1, & |c^i| \geq \lambda^i \\ 0, & |c^i| < \lambda^i \end{cases}
\]

(5)

where, \(\lambda^i\) is the exponential threshold for the \(i\)-th iteration, and it is expressed as

\[
\lambda^i = \text{Max} \cdot e^{\left(\ln(c) - \ln(\text{Max})\right)/M} 
\]

(6)

where Max is the maximum absolute value of the curvelet coefficients and then value of epsilon is close to zero. The reader is referred to Zhang and Chen (2013) for the detail threshold model selection. 

For regularly missing data, the conventional POCs method defined by eq. (4) fails to reconstruct the missing traces. The reason is that regular undersampling gives rise to well-known aliasing artefacts that look like the original signal components. In this case, it is difficult to detect only the signal components. By incorporate the mask function \(H\) into the conventional POCs iteration, Eq. (4) can be rewritten as

\[
d^i(t, x) = y^{obs} + [I - S(t, x)]C^iTQFQ^+^{-1}H^E(t, x), \quad i = 1, 2, ..., M
\]  

(7)

The mask function can eliminate the wraparound aliasing artefacts caused by regular undersampling, and the true spectra can be quickly converged in each iteration process.

3. Synthetic examples

To assess the performance for the proposed anti-aliasing interpolation method, the signal-to-noise ratio (SNR) is defined as \(\text{SNR} = 20 \log_{10} ||f||_2/||f - f^\text{interpolated}||_2\) (dB), where \(f^\text{model}\) represents the model data, and \(f^\text{interpolated}\) represents the interpolation data. The higher the values of SNR, the closer the interpolated results are to the model data.

In the following, a four-layer model is created to examine the performance of the proposed method. The synthetic data in the t-x and f-k domains is shown in Figs. 2(a) and (c), respectively. The original data contains 256 traces with 1024 samples per trace. The trace space and time sampling rate are 4 m and 2 ms, respectively. Fig. 2b shows a section of seismic data after regularly eliminating 50% of the traces. Fig. 2d shows the f-k spectrum of the section with missing traces. It is clear that the regular undersampling has produced replicas of the original spectrum of data, and it is very difficult to recover the missing traces if a method does not have anti-aliasing ability. We used curvelet transform with 6 scales and 16 angles at the 2nd coarsest scale in synthetic example. Firstly, the conventional POCs method is employed to interpolate the regularly missing traces. In order to get a better interpolation effect, the maximum iteration number for POCs interpolation is set to 40.

Fig. 3a shows its interpolated data, and the interpolated SNR is 18.67 dB. Figs. 3c and e show its f-k spectrum and the difference plot, respectively. From Figs. 3a and c, we can see that regularly missing traces have been recovered to some extent, but the performance is still unsatisfactory. Small artefacts are still visible in the interpolated data from the f-k spectrum, and local errors of difference plot are relatively large. Next, the AAPOCs method is applied to interpolate the missing traces in Fig. 2b. The maximum iteration number is set to 15. Figs. 3b and d show the t-x and f-k domains of the interpolated data using the proposed method, respectively, and the corresponding SNR is 21.34 dB, which is approximately 3 dB larger than that obtained using the conventional POCs. Clearly, the missing traces have been better restored successfully. In comparison to the two f-k spectra shown by Figs. 3c and d, respectively, we see the spectrum obtained by the AAPOCs interpolation approximately fits the spectrum of the original data, and it does not show repeated spectra, caused by aliasing artefacts. In addition, the residual errors in Fig. 3f are smaller in comparison to the errors given by the conventional POCs interpolation. We can conclude that the proposed method is more effective in anti-aliasing interpolation.

In order to further compare the performance, the recovered SNRs are compared as shown in Fig. 4, which illustrates that the recovered SNRs increase with iterations for two kinds of interpolation methods under the 50% regularly missing traces. It can be seen that the recovered SNRs of the AAPOCs method are always higher than that of the conventional POCs method under the same number of iterations. Moreover, the proposed method can achieve ideal interpolation results with only 12 iterations, and little is gained beyond that number of iterations. Therefore, we just select 15 iterations in the synthetic example as too many iterations require more time. On the contrary, the conventional POCs needs 40 iterations to get a relatively high SNR at least, and the highest SNR is only 18. 75 dB. In order to get this highest SNR, the AAPOCs method requires only 12 iterations, which shows that the proposed method has a fast convergence speed as well as high interpolation accuracy.

Now we show step by step how to obtain the results provided in Fig. 3b. Using the procedure given in Eq. (2), Fig. 5 shows the plot of E \((p)\) function computed from Fig. 2d for the dip range \(-5 \leq p \leq 5\) with the sampling interval of 0.08. One can clearly detect that there are many distinctive peaks in Fig. 5 that indicate the dip information of the stack.
curve events in the original data. However, we are more concerned about two boundary peak values on the left and right side because the peak values between the two boundaries represent dominant energy. After getting the two boundary peaks in Fig. 5, we expanded the position of two boundary peak values properly to preserve small spectral values of these events around the boundary dips, as shown in two red circles in Fig. 5. Based on these dips between two red circles, the mask function can be built via a suitable threshold using Eq. (3), as shown in Fig. 6. It is clear that this mask function accurately matches the f-k spectra of the original data, then we incorporate the mask function into the conventional POCS iteration in Eq. (4). During each iteration, the mask function can accurately converge the true f-k spectrum of the original signal, and eliminate the unwanted artefacts. Therefore, the proposed method can improve efficiency as well as keep interpolation accuracy.

In order to further examine the anti-aliasing ability of this proposed method, we decimate the original data by a factor of 4 and 6 (eliminate three and five traces between each pair of traces) to obtain the regularly missing data in Figs. 7a and b, respectively. Figs. 7c and d show the f-k spectra of data in Figs. 7a and b, respectively. It is clear that the regular undersampling has produced severely replicas of the original spectrum of data, and the values between the real spectrum and aliasing are approximately equal in the Fourier domain. Figs. 8a and b show the plot of $E(p)$ function computed from Figs. 7c and d for the dip range $-5 \leq p \leq 5$ using Eq. (2). Figs. 9a and b show the interpolated result from the Figs. 7a and b using the AAPCOS method, and the corresponding SNR is 17.58 dB and 14.76 dB, respectively. We can see that the missing traces has been restored successfully. The spatial aliasing effects disappear after interpolation in the f-k domain (Figs. 9c-d), and the interpolated dominant energy is consistent with the original signal. In contrast, the interpolation effect is relatively poor in Fig. 9b. It shows that the interpolation performance cannot reach the ideal satisfaction when there are severe missing traces. It is necessary to develop a high-dimensional AAPCOS method, and interpolates missing traces from another dimension.

The proposed method can also handle purely irregularly sampled data, and the $E(p)$ function and the mask function $H$ are estimated via a strategy that is same to that used for the regular undersampling case. Fig. 10a shows a section of seismic data after randomly eliminating 50% of the traces. Fig. 10c shows the f-k spectrum in Fig. 10a. We can see that irregular sampling attenuates coherent aliasing and produces a

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Fig. 9. Interpolated result and the f-k spectra. (a-b) Interpolated result from Fig. 7a and b, respectively. (c-d) The f-k spectra of (a and b), respectively.
noisy f-k spectrum. Figs. 10b and d show the t-x and f-k domains of the interpolated result using the proposed method, respectively, and its corresponding SNR is 22.07 dB. Clearly, the irregular missing traces have been restored successfully. The recovered SNR also demonstrates the validity of the proposed method.

4. Real data examples

Fig. 11a is a marine data, which contains 180 traces and 681 samples per trace. The trace interval is 25 m and the time sampling interval is 4 ms. Fig. 11c shows its f-k spectrum, and the data are spatially aliased above a normalized frequencies of 0.16. Fig. 11b shows a section of seismic data after regularly eliminating 50% of the traces, and Fig. 11d shows its f-k spectrum. Clearly, regular undersampling gives rise to well-known aliasing that look like the original signal components. We must remove the aliased energy that overlaps real aliasing-free energy. Figs. 12a and c show the interpolated data and its f-k spectra using the conventional POCs method, respectively. The difference plot is shown in Fig. 12e, and its corresponding SNR is 10.26 dB. We can see that regularly missing traces have been recovered to some extent, but small artefacts are still visible in the interpolated data from the f-k spectrum. The local errors are still relatively large. Then the AAPOCS method is proposed to interpolate the missing traces. Figs. 12b and d show the t-x and f-k domains of the interpolated data, respectively. The difference plot is shown in Fig. 12f, and its corresponding SNR is 12.15 dB. Clearly, the proposed method can eliminate aliasing more effectively, and the interpolated result show better continuity of events compared with the conventional POCs. Eventually, the proposed method obtain the satisfactory result which is consistent with the original data.

5. Discussion

The thresholding constant \( \tau \) was set with the following criteria: For synthetic data, we keep 40% of the largest amplitude coefficients. For the real data examples, we keep approximately 80% of the largest coefficients. This parameter is obtained with numerical tests in which we visually examined the f-k coefficients to decide for an optimal threshold constant. However, the thresholding constant \( \tau \) does not seem to be a critical parameter for our interpolation scheme, and it does not affect the interpolation result very much because another exponential threshold model is used in the iteration process, and some small amplitude coefficients can be also removed via the hard threshold operator. However, the thresholding constant \( \tau \) can accelerate its convergence speed. Meanwhile, through comparative simulation, we can find that the reconstruction effect of hard threshold is better than that of soft threshold. However, the main focus of this paper is not to present the threshold selection itself but focus on an anti-aliasing mechanism for POCs, so we don’t-test other thresholds.

It is easy to detect the two boundary peak values in function \( E(p) \) via setting a threshold value. However, the boundary peak value is smaller sometimes than other peak value. We need to get the distribution of energy for the dip range in which we can visually examine the function \( E \).
to decide the boundary peak value. Moreover, after getting the two boundary peak values, we need to expand their positions properly to preserve small spectral values of these events around the boundary dips. It can be implemented manually or empirically to decide how length should be expand for the slope $p$. In this paper, 12 sampling point of slope $p$ is suitable for the synthetic data according to the recovered SNR.

Even though Fourier transform-based anti-aliasing interpolation methods can handle curved events by analyzing the data in small spatial windows. It is always unreachable to get a ideal interpolation result, especially for the large gap missing traces. One of the advantage of this proposed method is that it can deal with curved event directly with a satisfactory interpolation accuracy. Moreover, The algorithm converges quite quickly because the mask function is introduced into the conventional POCS method, and satisfactory interpolation result can be obtained by 10–12 iterations.

6. Conclusions

In this paper, we proposed a novel strategy that utilizes conventional POCS method to interpolate regularly undersampled seismic data via the curvelet transform. Firstly, the curvelet transform was decomposed into f-k operator and the curvelet tiling operator. In the f-k domain, an angular search is carried out to identify the boundary of dominant dips, not only using low frequencies but over the whole frequency range. Second, we appropriately expand the position of two boundary dips to design a mask function in the f-k domain via a threshold strategy. This mask function can eliminate the wraparound aliasing artefacts caused by regular undersampling. Finally, by incorporating the mask function into conventional POCS method, we are able to derive a robust AAPCOS interpolation using the curvelet transform. The synthetic and real data tests illustrate that the proposed method can not only interpolate regularly undersampled seismic data, but also can effectively interpolate randomly undersampled seismic data. Moreover, the algorithm presented in this paper can be easily extended to deal with 3D spatial interpolation problems.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.
Fig. 12. Interpolated result of real seismic data. (a-b) Interpolated result using the POCS method and AAPOCS method, respectively. (c-d) The f-k spectra of (a-b), respectively. (e) and (f) are difference plots at the same scale as (a) and (b).
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